5E-1. $\int \frac{dx}{(x-2)(x+3)}dx$ 5E-2. $\int \frac{xdx}{(x-2)(x+3)}dx$ 5E-3. $\int \frac{xdx}{(x^2-4)(x+3)}dx$

5E-4. $\int \frac{3x^2+4x-11}{(x^2-1)(x-2)}dx$ 5E-5. $\int \frac{3x+2}{x(x+1)^2}dx$ 5E-6. $\int \frac{2x-9}{(x^2+9)(x+2)}dx$

3|9|25

2. $\int \frac{x}{(x-2)(x+3)}dx$

$\frac{2}{2+3} = A \Rightarrow A = \frac{2}{5}$

$\frac{-3}{-3-2} = B \Rightarrow B = \frac{3}{5}$

$= \int \frac{A}{x-2} + \frac{B}{x+3} dx$

$= \int \frac{2}{5(x-2)} + \frac{3}{5(x+3)} dx$

$= \frac{2}{5}\ln|x-2| + \frac{3}{5}\ln|x+3| + C$

3. $\int \frac{x}{(x^2-4)(x+3)}dx$

$\frac{-2}{(-2-2)(-2+3)} = A \Rightarrow A = \frac{1}{2}$

$= \int \frac{x}{(x+2)(x-2)(x+3)}dx$

$\frac{2}{(2+2)(2+3)} = B \Rightarrow B = \frac{1}{10}$

$\frac{-3}{(-3+2)(-3-2)} = C \Rightarrow C = -\frac{3}{5}$

$= \int \frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{x+3} dx$

$= \int \frac{1}{2(x+2)} + \frac{1}{10(x-2)} - \frac{3}{5(x+3)} dx$

$= \frac{1}{2}\ln|x+2| + \frac{1}{10}\ln|x-2| - \frac{3}{5}\ln|x+3| + C$

5. $$\int \frac{3x+2}{x(x+1)^2}\,dx$$

$$= \int \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\,dx$$

$$= \int \frac{2}{x} - \frac{2}{x+1} + \frac{1}{(x+1)^2}\,dx$$

$$= 2\ln|x| - 2\ln|x+1| - \frac{1}{x+1} + C$$

$$\frac{3(0)+2}{(0+1)^2} = A \Rightarrow A = 2$$

$$\frac{3(-1)+2}{-1} = C \Rightarrow C = 1$$

$$3x+2 = A(x+1)^2 + Bx(x+1) + C$$

$$0 = (A+B)x^2$$

$$\Rightarrow A+B=0$$

$$B = -2$$

6. $$\int \frac{2x-9}{(x^2+9)(x+2)}\,dx$$

$$= \int \frac{A}{x+2} + \frac{Bx+C}{x^2+9}\,dx$$

$$= \int -\frac{1}{x+2} + \frac{x}{x^2+9}\,dx$$

$$= -\ln|x+2| + \frac{1}{2}\ln|x^2+9| + C$$

$$\frac{2(-2)-9}{(-2)^2+9} = A \Rightarrow A = \frac{-4-9}{13}$$

$$= -1$$

$$2x-9 = A(x^2+9) + (Bx+C)(x+2)$$

$$x=0 \Rightarrow -9 = 9A + 2C$$

$$2C = -9+9$$

$$C = 0$$

$$2x = 2Bx \Rightarrow B = 1$$

$$\int \frac{x}{x^2+9}\,dx \qquad u = x^2+9 \Rightarrow du = 2x\,dx$$

$$= \int \frac{1}{2u}\,du$$

$$= \frac{1}{2}\ln|u|$$

$$= \frac{1}{2}\ln|x^2+9|$$

5E-10 Evaluate the following integrals

a) $\displaystyle\int \frac{dx}{x^3 - x}$ b) $\displaystyle\int \frac{(x+1)dx}{(x-2)(x-3)}$ c) $\displaystyle\int \frac{(x^2+x+1)dx}{x^2+8x}$

d) $\displaystyle\int \frac{(x^2+x+1)dx}{x^2+8x}$ e) $\displaystyle\int \frac{dx}{x^3+x^2}$ f) $\displaystyle\int \frac{(x^2+1)dx}{x^3+2x^2+x}$

g) $\displaystyle\int \frac{x^3 dx}{(x+1)^2(x-1)}$ h) $\displaystyle\int \frac{(x^2+1)dx}{x^2+2x+2}$

h) $$\int \frac{(x^2+1)}{x^2+2x+2}\,dx$$

$$= \int 1 + \frac{-2x-1}{x^2+2x+2}\,dx$$

$$= x - \ln\left|(x+1)^2+1\right| + \arctan(x+1) + C$$

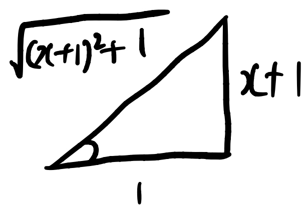
$$\begin{array}{r|l} & 1 \\ \hline x^2+2x+2 & x^2+0+1 \\ & x^2+2x+2 \\ \hline & -2x-1 \end{array}$$

$\tan u = x+1$

$x = \tan u - 1$

$dx = \sec^2 u\, du$

$$\int -\frac{2x+1}{x^2+2x+2}\,dx$$



$$= -\int \frac{2x+1}{(x+1)^2+1}\,dx$$

$$= -\int \frac{2\tan u - 2 + 1}{\tan^2 u + 1}\sec^2 u\,du$$

$$= -\int \frac{2\tan u - 1}{\sec^2 u}\sec^2 u\,du$$

$$= -\int 2\tan u - 1\,du$$

$$= -(-2\ln|\cos u| - u) + C$$

$$= 2\ln\left|\frac{1}{\sqrt{(x+1)^2+1}}\right| + \arctan(x+1) + C$$

$$= -\ln\left|(x+1)^2+1\right| + \arctan(x+1) + C$$

Evaluate the following integrals

5F-1 ~~a~~) $\int x^a \ln x dx$ $(a \neq -1)$ b) Evaluate the case $a = -1$ by substitution.

$u = \ln x$

$v' = x^a$

a) $\int x^a \ln x dx$ $(a \neq -1)$

$= \ln x \cdot \frac{x^{a+1}}{a+1} - \int \frac{1}{x} \cdot \frac{x^{a+1}}{a+1} dx$

$= \frac{x^{a+1} \ln x}{a+1} - \frac{1}{a+1} \cdot \frac{x^{a+1}}{a+1} + C$

$= \frac{x^{a+1}}{a+1}\left(\ln x - \frac{1}{a+1}\right) + C$ $(a \neq -1)$

2. ~~d~~) Derive the reduction formula expressing $\int x^n e^{ax}\,dx$ in terms of $\int x^{n-1}e^{ax}\,dx$.

5F-2 a) $\int xe^x\,dx$ ~~b~~) $\int x^2e^x\,dx$ c) $\int x^3e^x\,dx$

d) $\int x^n e^{ax}\,dx$ Let $F_n(x) = \int x^n e^{ax}\,dx$.

$u = x^n$

$v' = e^{ax}$

$$= x^n \cdot \frac{e^{ax}}{a} - \int n x^{n-1} \cdot \frac{e^{ax}}{a}\,dx$$

$$= \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$$

$$= \frac{1}{a}\left(x^n e^{ax} - n F_{n-1}(x)\right)$$

$$\therefore \int x^n e^{ax}\,dx = \frac{1}{a}\left(x^n e^{ax} - n\int x^{n-1}e^{ax}\,dx\right)$$

b) $$\int x^2e^x\,dx = x^2e^x - 2\int xe^x\,dx$$

$$= x^2e^x - 2\left(xe^x - \int e^x\,dx\right)$$

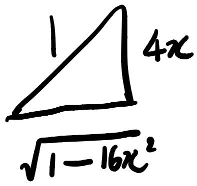
$$= x^2e^x - 2xe^x + 2e^x + C$$

5F-3 Evaluate $\int \sin^{-1}(4x)dx$

Let $u = \sin^{-1}4x$,

then $\sin u = 4x$

$dx = \frac{\cos u}{4} du$



$$\int \sin^{-1}(4x)\,dx$$

$$= \int u \cdot \frac{\cos u}{4} du$$

$$= u\left(\frac{\sin u}{4}\right) - \int 1 \cdot \frac{\sin u}{4} du$$

$$= \frac{u \sin u}{4} - \frac{(-\cos u)}{4} + C$$

$$= \frac{4x \sin^{-1}4x}{4} + \frac{\sqrt{1-16x^2}}{4} + C$$

$$= x \sin^{-1}4x + \frac{\sqrt{1-16x^2}}{4} + C$$